Cylindrical Shells of Variable Wall Thickness

Thomas J. Lardner*
Massachusetts Institute of Technology, Cambridge, Mass.

In two recent Notes on cylindrical shells of variable wall thickness,[1,2] generalized hypergeometric functions were used to obtain influence coefficients[1] and to obtain particular solutions for a class of normal pressure loading proportional to a power of the axial distance along the shell.[2] In a discussion of a related paper on hypergeometric functions,[3] it was noted that it is possible to obtain a class of exact solutions expressible in terms of Bessel functions for certain power variations of the wall thickness. This class of exact solutions has been presented in a recent paper.[4]

It is the purpose of this Comment to note that, for the class of exact solutions in Ref. 4, the particular solutions for the class of pressure loadings proportional to a power of the axial distance discussed in Ref. 2 can also be expressed in terms of Bessel functions. This observation can be confirmed easily by use of Eqs. (9) and (17) of Ref. 2 together with the results in Ref. 4. It is felt that the use of these exact solutions will provide an interesting complement to the direct numerical determination of the hypergeometric functions.

References

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* Assistant Professor of Mechanical Engineering.

Comments on "Compressible Turbulent Boundary-Layer Equations"

P. Bradshaw*
National Physical Laboratory, Teddington, England

Rubin[1] has suggested that certain discrepancies exist in the equation for compressible, turbulent boundary layers as quoted in the literature. Although some discrepancies do exist, as will be shown below, Rubin's results are chiefly the consequence of omitting from the enthalpy equation the term representing production of enthalpy by viscous dissipation of turbulent kinetic energy. Van Driest,[2] followed by Pai,[3] approximates this dissipation term by the rate of production of turbulent kinetic energy by shear forces at the expense of mean-flow kinetic energy, and to this approximation the static enthalpy equation is

$$\bar{K} + \overline{(\rho v)'u'}\bar{u}_y = 0, \tag{1}$$

using Rubin's notation, and not $\bar{K} = 0$ as stated by Rubin. Now Eq. (1) is exactly Rubin's Eq. (7), obtained by manipulating the other equations, so that Rubin's demonstration that the rate of turbulence production $\overline{(\rho v)'u'}.\bar{u}_y$ is zero follows mainly from his erroneous assumption that $\bar{K} = 0$, and not, as he suggests, entirely from the neglect of a third-order term $[\overline{(\rho v)'u'^2}/2]_y$. This term is indeed neglected by Van Driest, but so are several others. Van Driest, in effect, reduces the turbulent kinetic energy equation (which represents the rate of change of turbulent kinetic energy $\frac{1}{2}\bar{\rho}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})$ along a mean streamline) to "production" = "dissipation," whereas it is really "dilation production" + "shear production" = "dissipation" + "loss by diffusion" + "rate of increase along streamline" (in Rubin's symbolic notation, it is $\overline{uM} - \bar{u}\,\bar{M} = 0$). The term mentioned by Rubin is part of the diffusion: the equation is given in full by Bradshaw and Ferriss,[4] who also discuss the approximations permissible at nonhypersonic Mach numbers. The gross assumption "shear production" = "dissipation" is seldom in error by more than, say, 20%, and is quite accurate close to the surface where the terms are largest, so that it should suffice as an approximation to the heat-source term in the enthalpy equations. Thus, Van Driest's form of the compressible turbulent boundary-layer equations is restored as an engineering approximation.

References

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* Principal Scientific Officer, Aerodynamics Division.

Reply by Author to P. Bradshaw

Stanley G. Rubin*
Polytechnic Institute of Brooklyn—Graduate Center, Farmingdale, N. Y.

The result obtained previously by this author[1] was predicated on the postulate that the well-known boundary-layer approximation that involves one streamwise and one normal coordinate scale is applicable to the turbulent fluctuations as well as the mean quantities. As a consequence of this postulate, molecular fluctuations were not retained. However, if the production of turbulent energy, $-\overline{(\rho v)'u'}\bar{u}_y$ is, in fact, balanced primarily by a molecular dissipation of turbulent energy, typically $\bar{\mu}(u'_y)^2$, and only to a lesser degree by diffusion which includes triple correlation effects, then the hypothesis of Ref. 1 is not justified.

For low-speed flow, experimental measurement of the various fluctuations indicates that Bradshaw's comments are correct and that the length scale for fluctuation gradients is much smaller than that for the mean quantities. In this case, the Van Driest approximation should be in error by the neglect of diffusion and turbulent energy convection, or at

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* Associate Professor, Department of Aerospace Engineering.